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Quark condensate in nuclear matter based on Nuclear Schwinger-Dyson formalism

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abstract

The effects of higher order corrections of ring diagrams for the quark condensate are studied by using the bare vertex Nuclear Schwinger Dyson formalism based on σ-ω model. At the high density the quark condensate is reduced by the higher order contribution of ring diagrams more than the mean field theory or the Hartree-Fock .

The quantum chromodynamics (QCD) is a useful theory to describe quarks and gluons. Especially the perturbative QCD works very well at high energies. But at low energies the QCD shows non-perturbative behaviors. The QCD sum-rule approach is a useful tool in understanding the property of hadrons in free space [1]. The vacuum expectation value of the quark condensate is estimated by the sum-rule approach,

$$< \bar{q}q >_0 \simeq -(225 \pm 25\mathrm{MeV})^3. \tag{1}$$

The QCD is not yet available for hadronic phenomena , but there have been several quantum hadrodynamics approachs to describe the properties of hadrons in nuclear matter. The quark condensate, which is shifted from vacuum values, is determined using these approachs .

Recently the quark condensate in medium has been evaluated in the framework of mean field theory (MFT), Nambu-Jona-Lasinio (NJL) model [2], and relativistic Brueckner Hartree-Fock (RBHF) [3][4] . The results of these methods show that the quark condensate in nuclear matter is reduced considerably at the normal density. The Feynman-Hellmann theorem relates the shift of quark condensate from vacuum value to the pion-nucleon sigma term $\sigma_{\pi N}$ which can be related to the pion-nucleon scattering amplitude. Using the Feynman-Hellmann theorem the quark condensate $< \bar{q}q >_\rho$ in nuclear matter is related to energy density ϵ as follows,

$$2m_q(< \bar{q}q >_\rho - < \bar{q}q >_0) = m_q \frac{d\epsilon}{dm_q}, \tag{2}$$

where $< \bar{q}q >_\rho \equiv < \rho|\bar{q}q|\rho >$, and $< \bar{q}q >_0 \equiv < 0|\bar{q}q|0 >$. The energy density ϵ in nuclear

matter can be written as $\epsilon = (m_N + E/A)\rho$,where m_N is the nucleon mass in free space and E/A is the energy per nucleon in nuclear matter and ρ is the baryon density. If we neglect the dependence of meson-nucleon coupling constants on the current quark mass, and using the Gell-Mann-Oakes-Renner relation, $2m_q < \bar{q}q >_0 = -m_\pi^2 f_\pi^2$ (where m_π ≈138 MeV is the pion mass and $f_\pi \approx$ 93 MeV is the pion decay constant) and the definition of the pion-nucleon sigma term , $\sigma_{\pi N} = m_q(dm_N/dm_q)$, the quark condensate is obtained as following expression

$$\frac{< \bar{q}q >_\rho}{< \bar{q}q >_0} = 1 - \frac{\rho}{m_\pi^2 f_\pi^2}\left[\sigma_{\pi N} + m_q \frac{d}{dm_q}\left(\frac{E}{A}\right)\right]. \tag{3}$$

We accept $\sigma_{\pi N} =$ (45± 7) MeV which was recently analyzed by Gasser et al [5].

In this paper, we study the quark condensate in nuclear matter using the bare vertex Schwinger Dyson (BNSD) formalism [6][7] based on σ-ω model and compare the result of the BNSD with the those of the MFT and the HF , since our purpose is the examination of the contribution of the higher order ring diagrams for quark condensate. Since we don't consider the current quark mass dependence of coupling constants between a nucleon and a meson, the derivatives of these meson masses are approximately related to the nucleon mass [8] as follows,

$$\left(\frac{dm_i}{dm_q}\right) / \left(\frac{dm_N}{dm_q}\right) = \frac{m_i}{m_N}, (i = \sigma \quad and \quad \omega). \tag{4}$$

We acquire the final expression of the quark condensate in nuclear matter in σ-ω

model ,

$$\frac{<\bar{q}q>_\rho}{<\bar{q}q>_0} = 1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho\left[1 + \frac{\partial(E/A)}{\partial m_N} + \sum_{i=\sigma,\omega}\frac{\partial(E/A)}{\partial m_i}\frac{m_i}{m_N}\right]. \tag{5}$$

To evaluate the quark condensate in medium we need the energy density of the nuclear matter. But the direct results from the QCD is not obtainable and so alternatively we calculate the energy density in nuclear matter using the BNSD in σ-ω model. This one modifies the Fock exchange self-energy in the HF by replacing free meson propagators with full ones which are accompanied with the particle-hole excitations. We adopt the Walecka model[9] which consists of three fields, the nucleon ψ, the scalar σ-meson ϕ and the vector ω-meson V_μ . The lagrangian density is given by

$$L = -\bar{\psi}(\gamma_\mu\partial_\mu + m_N)\psi - \frac{1}{2}(\partial_\mu\phi\partial_\mu\phi + m_s^2\phi^2)$$

$$-(\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_v^2 V_\mu V_\mu) + g_s\bar{\psi}\psi\phi + ig_v\bar{\psi}\gamma_\mu\psi V_\mu, \tag{6}$$

where $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and m_N, m_s, m_v, g_s and g_v are nucleon mass, σ meson mass, ω meson mass, σ-nucleon and ω-nucleon coupling constants respectively. The nucleon propagator is obtained by following form,

$$G(k) = G^0(k) + G^0(k)\Sigma(k)G(k), \tag{7}$$

where Σ is nucleon self-energy. The mesons full propagators for the scalar and vector mesons are obtained as follows [6] [7],

$$D_{ab}(k) = D_{ab}^0(k) + D_{ac}^0(k)\Pi_{cd}(k)D_{db}(k), \tag{8}$$

where D_{ab} and Π_{cd} are expressed in terms of 5×5 matrices taking into account the

mixture of σ and ω meson in nuclear matter. Meson propagator D_{ab} is obtained by solving the Dyson equation

$$D_{ab}(k) = \begin{pmatrix} D_{\mu\nu} & D_{5\nu} \\ D_{\mu 5} & D_s \end{pmatrix}, \tag{9}$$

$$D_{\mu\nu} = \delta_{\mu\nu} D_l(k) + \frac{k_\mu k_\nu}{k_\rho^2} \Big\{ D_0(k) \Big(1 + \frac{k_\lambda^2}{m_v^2} \Big) - D_l(k) \Big\}$$

$$+ \Big(\delta_{ij} - \frac{k_i k_j}{k^2} \Big) (D_t(k) - D_l(k)) \delta_{\mu i} \delta_{\nu j}, \tag{10}$$

$$D_{5\nu}(k) = D_{\mu 5}(k) = \begin{cases} i \frac{k_i k_4}{k_\rho^2} D_m(k) & for \qquad \mu = i = 1 \sim 3 \\ -i D_m(k) & for \qquad \mu = 4 \end{cases}, \tag{11}$$

where D_0 denotes the non-interacting meson propagator. The subscripts s, l, t and m of $D(k)$ denote the component of σ meson, the longitudinal and transverse component of ω meson and the component of mixture of σ and ω mesons, respectively. Eqs.(7) and (8) are solved self-consistently.

The total energy density is given in six parts as follows,

$$\epsilon = \epsilon_B + \epsilon_{C,\sigma} + \epsilon_{C,\omega} + \epsilon_{SD,\sigma} + \epsilon_{SD,\omega} + \epsilon_{SD,m}, \tag{12}$$

where subscripts B, C and SD denote the baryon part, the classical part and the quantum part, respectively. The details of the energy densities of these components are as follows,

$$\epsilon_B = \frac{\lambda}{\pi^2}\int_0^{k_F} q^2 dq E_q, \tag{13}$$

$$\epsilon_{C,\sigma} = \frac{1}{2}\left(\frac{g_s}{m_s}\right)^2 \rho_s^2, \qquad \rho_s = \frac{\lambda}{\pi^2}\int_0^{k_F} k^2 dk \frac{M_k^*}{E_k^*}, \tag{14}$$

$$\epsilon_{C,\omega} = -\frac{1}{2}\left(\frac{g_v}{m_v}\right)^2 \rho_B^2, \qquad \rho_B = \frac{\lambda}{3\pi^2} k_F^3, \tag{15}$$

$$\epsilon_{SD,\sigma} = \frac{\lambda g_s^2}{16\pi^4}\int_0^{k_F} \frac{q^2 dq}{E_q^*}\int_0^{k_F} \frac{k^2 dk}{E_k^*}\int_{-1}^{1} dx\{1 - 2(E_k - E_q)^2 D_s^0(R)\}$$

$$\times\, (M_k^* M_q^* - q^* k^* x + E_q^* E_k^*) D_s(R), \tag{16}$$

$$\epsilon_{SD,\omega} = -\frac{\lambda g_v^2}{16\pi^4}\int_0^{k_F} \frac{q^2 dq}{E_q^*}\int_0^{k_F} \frac{k^2 dk}{E_k^*}\int_{-1}^{1} dx\{1 - 2(E_k - E_q)^2 D_v^0(R)\}$$

$$\times\Bigg(4(M_k^* M_q^* + 2k_\mu^* q_\nu^*) D_l(R) + \Bigg\{\left(4 - \frac{R_\mu^2}{R^2}\right) M_q^* M_k^*$$

$$+\Bigg\{\left(2 - \frac{R_\mu^2}{R^2}\right) q^* k^* x - \left(2 + \frac{R_\mu^2}{R^2}\right) E_q^* E_k^*\Bigg\}(D_t(k) - D_l(k))\Bigg), \tag{17}$$

$$\epsilon_{SD,M} = \frac{\lambda g_s g_v}{8\pi^4}\int_0^{k_F} \frac{q^2 dq}{E_q^*}\int_0^{k_F} \frac{k^2 dk}{E_k^*}\int_{-1}^{1} dx\{1 - (E_k - E_q)^2 (D_s^0(R) + D_v^0(R)\}$$

$$\times\, \frac{R_\mu^2}{R^2}(E_q^* M_k^* + E_k^* M_q^*) D_m(R), \tag{18}$$

where E_k is a spectrum of the nucleon in the nuclear matter and $E^*_k = \sqrt{\vec{k}^{*2} + M^{*2}_k}$ and $M^*_k = M + \Sigma_s(k)$, $R_\mu = k_\mu - q_\mu$ and λ denotes the degeneracy, $\lambda = 2$ for nuclear matter and $\lambda = 1$ for neutron matter. If we neglect the quantum parts of Eqs. (16) $\sim$ (18), the expressions correspond to the MFT. Similarly if one replace the full meson propagator D with the free one D^0 and neglect the mixture D_m, one obtain the HF .

We neglect the other nonstrange mesons $(\pi, \eta, \delta$, and $\rho)$ because the contributions of these mesons for quark condensate are much smaller than the contributions of σ and ω mesons [3] [4].

In Fig.1, we show the quark condensate in nuclear matter calculated by some methods. The dotted line denotes the results in the leading order approximation. The solid line, dashed line, and dot-dashed line denote the results based on the BNSD , the MFT and the HF , respectively. The scalar and vector coupling constants are chosen to satisfy the boundary condition at the normal density of nuclear matter (ρ_0 =0.17fm^{-3}) for each methods. In the BNSD, the HF and the MFT , the quark condensate tends to increase around 1.5 times of the normal density, contrary to expectations based on ideas of chiral symmetry restoration. Comparing the result of the BNSD with the one of the HF, we notice the higher order ring correlation reduces the quark condensate.

The detail of this reduction of quark condensate is shown in Fig.2. In this figure we show the density dependence of derivative of the energy per nucleon with respect to the nucleon mass, the σ meson mass and the ω meson mass. The solid line and dot-dashed line denote the results of the BNSD and the HF, respectively. The contribution from

the σ meson in the BNSD is larger than the one in the HF by the correction of the higher order diagrams . Similarly the contribution from the ω meson in the BNSD is smaller than the one in the HF by the correction of the higher order diagrams. While the contribution from the nucleon in the BNSD is nearly equal to the one in the HF. The higher order contribution from the nucleon mass derivative is hardly noticeable. The contributions from the nucleon, the σ meson and the ω meson cancel out one another. The net contribution in the cancellation reduces the quark condensate at the vacuum value. Since the net contribution in the BNSD is larger than the one in the HF , the quark condensate in the BNSD is reduced more than the one in the HF.

In summary, we have calculated the quark condensate in nuclear matter using the BNSD and found that the quark condensate at the normal density is reduced about 35% compared to the one at zero density in the BNSD as same as in the MFT and the HF (see Fig.1). The difference of these three models appears at high densities. The higher order contribution of ring diagrams dose not affect the quark condensate at low densities. But the contribution is important at high densities.

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Figure caption

Fig.1 : The quark condensate in nuclear matter. The dotted line denotes the results in the leading order approximation. The solid line, dashed line, and dot dash line denote the results based on the BNSD, the MFT and the HF , respectively.

Fig.2 : Derivatives of the energy-per-nucleon ($\partial(E/A)/\partial m_i$) in nuclear matter. The solid line and dot dash line denote the results based on the BNSD and the HF.